كلية التقنية الالكترونية طرابلس
College of Electronic Technology - Tripoli

القســـــم: الاتصالات والتحكم | أسئلة الامتحان النصفي الثاني لمادة: الإشارات والنظم
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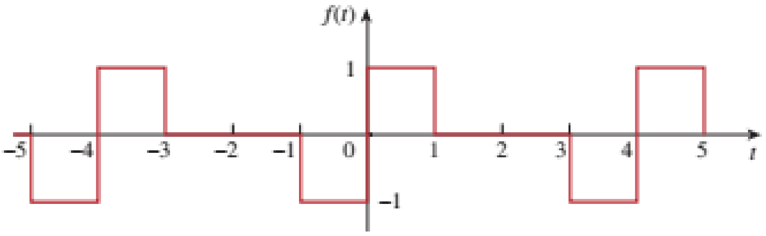
Question 1:

a. Compute and sketch the spectra of the first 5 components of the trigonometric Fourier series for the waveform shown below. ***[8]***



b. A signal x(t) has a Fourier series expansion as:

$$x(t) = \sum_{n=0}^{\infty} \left[\frac{8}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right)\right] \sin\left(\frac{n\pi}{2} t\right)$$

Sketch Double Side magnitude and phase line spectra of x(t) showing all important values on the sketches. ***[5]***

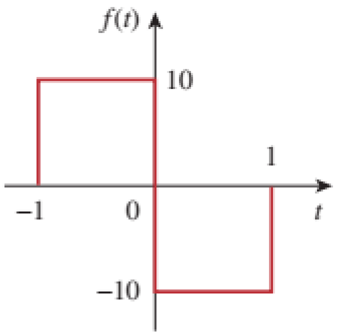
Question 2:

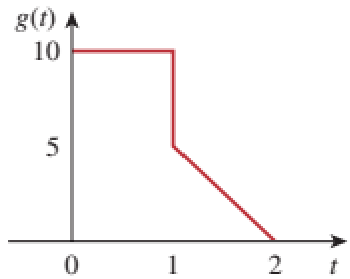
a. Use the Fourier integral directly to find the transform of each of the following signals:

i. $x(t) = \delta(t) - \delta(t-2)$ ***[3.5]***

ii. $f(t) = e^{-2t}[u(t) - u(t-6)]$ ***[3.5]***

b. Using tables of Fourier transforms and properties, Determine the Fourier transform of the following signals: ***[10]***





Transformations' table:

$x(t)$	$X(\omega)$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$sgn(t)$	$\frac{2}{j\omega}$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$P_a(t) = \begin{cases} A & \lvert t\rvert < a \\ 0 & \lvert t\rvert > a \end{cases}$ Pulse Duration=2a Or $A\,rect(\frac{t}{2a})$	$2Aa\frac{\sin(\omega a)}{\omega a}$
$te^{-at}u(t)$	$\frac{1}{(j\omega+a)^2}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$

Properties' table:

Property	Time Domain	Fourier Domain
Time shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Time differentiation	$\frac{dx}{dt}$	$j\omega X(\omega)$
Frequency differentiation	$-tx(t)$	$-j\,\frac{dX}{d\omega}$
Duality	$X(t)$	$2\pi X(-\omega)$
Time convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$

مع تمنياتي للجميع بالتوفيق والنجاح

Midterm exam solution :-

Q1 :- (a)

By inspection $f(t)$ is an odd signal.

$a_0 = a_n = 0$

$T_0 = 4(s)$, $\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/s

$$f(t+T_0) = \begin{cases} -1, & -1 < t \leq 0 \\ 1, & 0 \leq t \leq 1 \end{cases}$$

$$b_n = \frac{2}{T}\left[\int_{-1}^{0} -\sin(n\omega t)\,.dt + \int_{0}^{1} \sin(n\omega t)\,.dt\right]$$

$$b_n = \frac{2}{n\omega T}\left[\cos(n\omega t)\Big|_{-1}^{0} - \cos(n\omega t)\Big|_{0}^{1}\right]$$

$$b_n = \frac{2}{2\pi n}\left[\overset{1}{\cos(0)} - \cos\left(\overset{\text{even}}{-\frac{n\pi}{2}}\right) - \cos\left(\frac{n\pi}{2}\right) + \overset{1}{\cos(0)}\right]$$

$$b_n = \frac{1}{n\pi}\left[2 - 2\cos\left(\frac{n\pi}{2}\right)\right]$$

$$b_n = \frac{2}{n\pi}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]$$

$$f(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{1}{n}\left(1 - \cos\left(\frac{n\pi}{2}\right)\right)\sin\left(\frac{n\pi}{2}t\right)\right]$$

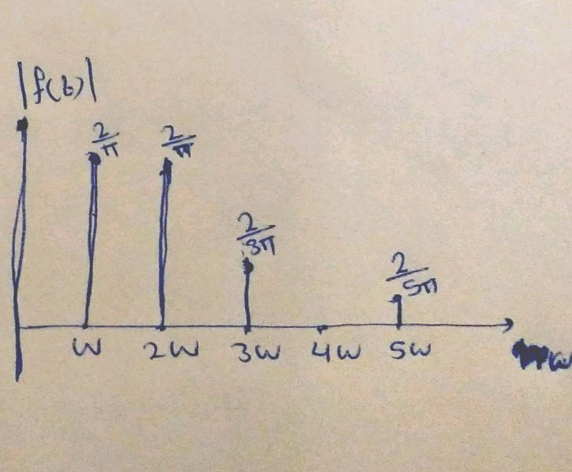
$$b_1 = \frac{2}{\pi}\left[1 - \cos\left(\frac{\pi}{2}\right)\right] = \frac{2}{\pi}$$

$$b_2 = \frac{2}{2\pi}\left[1 - \cos(\pi)\right] = \frac{1}{\pi}\left[1+1\right] = \frac{2}{\pi}$$

$$b_3 = \frac{2}{3\pi}\left[1 - \cos\left(\frac{3\pi}{2}\right)\right] = \frac{2}{3\pi}$$

$$b_4 = \frac{2}{4\pi}\left[1 - \cos(2\pi)\right] = 0$$

$$b_5 = \frac{2}{5\pi}\left[1 - \cos\left(\frac{5\pi}{2}\right)\right] = \frac{2}{5\pi}\left[1 - 0\right] = \frac{2}{5\pi}$$



(b) $x(t) = \sum_{n=1}^{\infty} \left[\frac{8}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right)\right] \sin\left(\frac{n\pi}{2} t\right)$

By comparing to the trigonometric Fourier series "

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right]$$

It can be noticed that $a_0 = a_n = 0$

So it is odd signal.

$\omega = \frac{\pi}{2}$

$b_n = \frac{8}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right)$

$b_1 = \frac{8}{\pi^2} \sin\left(\frac{\pi}{2}\right) = 0.81$

$b_2 = \frac{8}{4\pi^2} \sin(\pi) = 0$

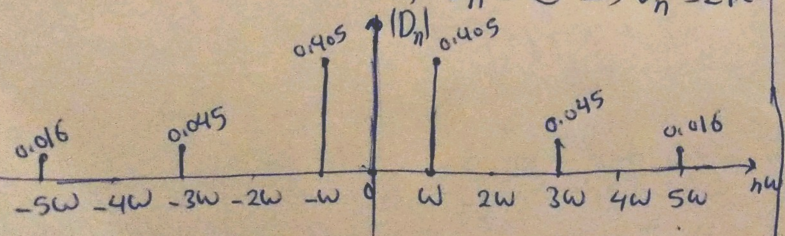
$b_3 = \frac{8}{9\pi^2} \sin\left(\frac{3\pi}{2}\right) = -0.09$

$b_4 = \frac{8}{16\pi^2} \sin(2\pi) = 0$

$b_5 = \frac{8}{25\pi^2} \sin\left(\frac{5\pi}{2}\right) = 0.032$

In general to draw the double side spectrum from odd signal component (b_n) devide by $\underline{\underline{2}}$ the phase "angle" is always either 90° or 270° depending on the sign of b_n ⇒ if $b_n = \oplus \Rightarrow \theta_n = 90^\circ$

if $b_n = \ominus \Rightarrow \theta_n = 270^\circ$



$D_n = \frac{a_n - jb_n}{2}$

$D_1 = \frac{-j0.81}{2} = -j0.405$

$D_2 = 0$

$D_3 = \frac{j0.09}{2} = j0.045$

$D_4 = 0$

$D_5 = \frac{-j0.032}{2} = -j0.016$

$|D_1| = 0.405$, $\theta_1 = \tan^{-1}\left(\frac{-0.405}{0}\right)$

$= 270^\circ$ or -90°

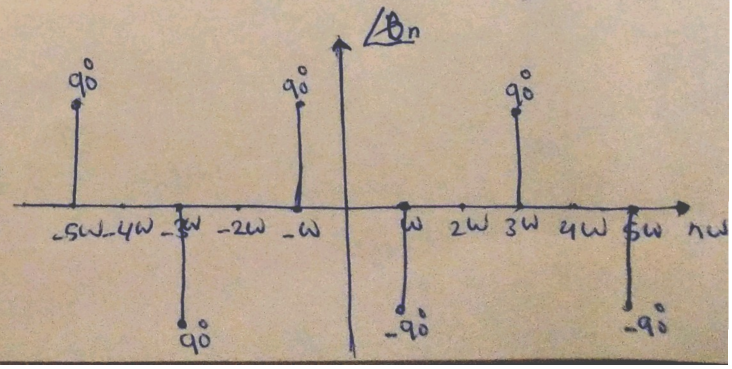
$|D_2| = 0$ ⇒ no 2nd harmonics.

$|D_3| = 0.045$ ⇒ $\theta_3 = \tan^{-1}\left(\frac{0.045}{0}\right) = 90^\circ$

$|D_4| = 0$ ⇒ no 4th harmonic.

$|D_5| = 0.016$, $\theta_n = \tan^{-1}\left(\frac{-0.016}{0}\right)$

$= -90^\circ$, 270°



Q2

a) (i) $x(t) = \delta(t) - \delta(t-2)$.

$$X(\omega) = \int_{-\infty}^{\infty} \underbrace{\delta(t)}_{t_0=0} e^{-j\omega t}\,dt - \int_{-\infty}^{\infty} \underbrace{\delta(t-2)}_{t_0=2} e^{-j\omega t}\,dt \qquad X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt$$

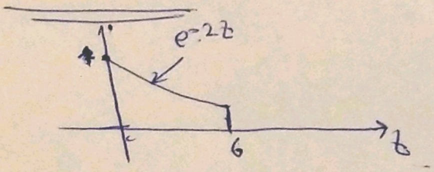
Using ~~the~~ integration property of unit impulse.

$$X(\omega) = e^{0} - e^{-j\omega(2)} = 1 - e^{-2j\omega}$$

* To confirm from table $\delta(t) \xrightarrow{F.T} 1$

$$-\delta(t-2) \Longrightarrow -(1).\,e^{-2j\omega} = -e^{-2j\omega}$$

(ii) $f(t) = e^{-2t}[u(t) - u(t-6)]$



$$F(\omega) = \int_0^6 e^{-2t}.\,e^{-j\omega t}\,dt$$

$$F(\omega) = \int_0^6 e^{-(2+j\omega)t}\,dt$$

$$F(\omega) = \frac{-1}{2+j\omega}\, e^{-(2+j\omega)t}\Big|_0^6 = \frac{-1}{2+j\omega}\left[e^{-6(2+j\omega)} - e^{0}\right]$$

$$F(\omega) = \frac{1}{2+j\omega}\left[1 - e^{-12}.\,e^{-6j\omega}\right]$$

* To confirm from table:-

$$e^{-2t}u(t) \xrightarrow{F.T} \frac{1}{2+j\omega}$$

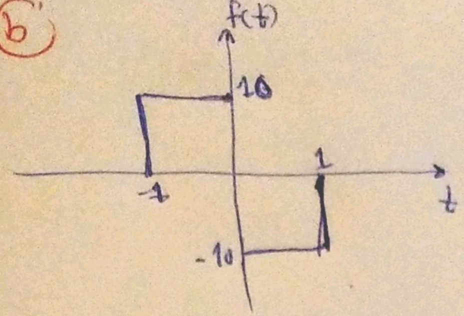
$- e^{-2t} u(t-6)$ * Remember that the shift of $u(t)$ must be the same with all other t's in the signal.

$$-e^{-2(t-6+6)} u(t-6) = -e^{-2(t-6)}.\,e^{-12} \leftarrow \text{constant}$$

$$-e^{-12}.\,e^{-2(t-6)} \text{ time shift} \xrightarrow{F.T} -e^{-12}.\,\frac{1}{2+j\omega}\, e^{-6j\omega}$$

Q2 (b)

(i)



$$f(t) = 10\,\text{rect}\left(t+\tfrac{1}{2}\right) - 10\,\text{rect}\left(t-\tfrac{1}{2}\right)$$

$$F(\omega) = 10\,\frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}\,e^{+j\frac{\omega}{2}} - 10\,\frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}\,e^{-j\frac{\omega}{2}}$$

$$F(\omega) = 20\,\frac{\sin\left(\frac{\omega}{2}\right)e^{+j\frac{\omega}{2}}}{\omega} - 20\,\frac{\sin\left(\frac{\omega}{2}\right)e^{-j\frac{\omega}{2}}}{\omega}$$

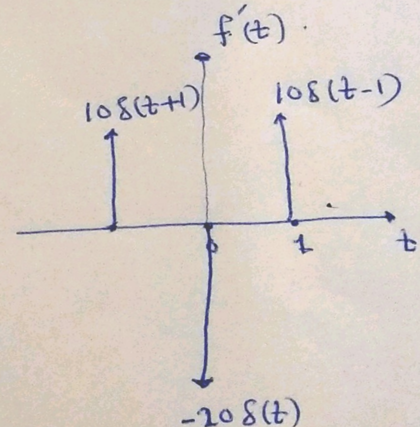
$$F(\omega) = \frac{20}{\omega}\left[\frac{\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)e^{j\frac{\omega}{2}}}{2j} - \frac{\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)e^{-j\frac{\omega}{2}}}{2j}\right]$$

$$F(\omega) = \frac{10}{j\omega}\left[e^{j\omega} - 1 - 1 + e^{-j\omega}\right]$$

$$F(\omega) = \frac{10}{j\omega}\left[e^{j\omega} - e^{-j\omega} - 2\right]$$

$$F(\omega) = \frac{20}{j\omega}\left[\frac{e^{j\omega} - e^{-j\omega}}{2} - \frac{2}{2}\right] = \frac{20}{j\omega}\left[\cos(\omega) - 1\right]$$

Also :- $f(t) = 10\,u(t+1) - 20\,u(t) + 10\,u(t-1)$



$$f'(t) = 10\,\delta(t+1) - 20\,\delta(t) + 10\,\delta(t-1)$$

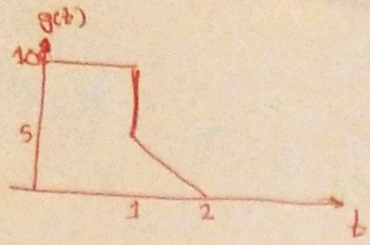
$$(j\omega)F(\omega) = 10e^{j\omega} - 20 + 10e^{-j\omega}$$

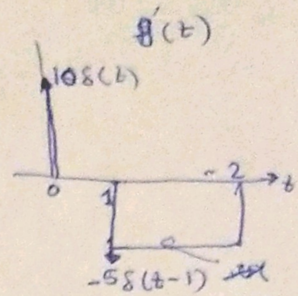
$$F(\omega) = \frac{10e^{j\omega} + 10e^{-j\omega} - 20}{j\omega}$$

$$F(\omega) = \frac{10}{j\omega}\left[\frac{2\left(e^{j\omega} + e^{-j\omega}\right)}{2} - 2\right]$$

$$F(\omega) = \frac{20}{j\omega}\left[\cos(\omega) - 1\right]$$

(ii)

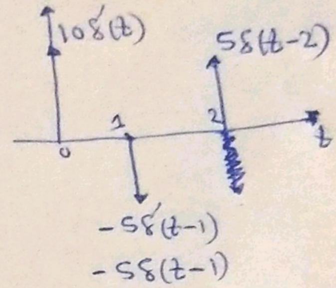




$$g(t) = 10u(t) - 5u(t-1) - 5r(t-1) + 5r(t-2)$$

$$g'(t) = 10\delta(t) - 5\delta(t-1) - 5u(t-1) + 5u(t-2)$$

$$g''(t) = 10\delta'(t) - 5\delta'(t-1) - 5\delta(t-1) + 5\delta(t-2)$$



$$(j\omega)^2 G(\omega) = 10j\omega - 5j\omega e^{-j\omega} - 5e^{-j\omega} + 5e^{-2j\omega}$$

$$G(\omega) = \frac{-10j\omega - 5e^{-2j\omega} + 5j\omega e^{-j\omega} + 5e^{-j\omega}}{\omega^2}$$

$$G(\omega) = \frac{5}{\omega^2}\left[-2j\omega - e^{-2j\omega} + j\omega e^{-j\omega} + e^{-j\omega}\right]$$